

Fermion scattering off electroweak phase transition kink walls with hypermagnetic fields

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We study the scattering of fermions off a finite width kink wall during the electroweak phase transition in the presence of a background hypermagnetic field. We derive and solve the Dirac equation for such fermions and compute the reflection and transmission coefficients for the case when the fermions move from the symmetric to the broken symmetry phase. We show that the chiral nature of the fermion coupling with the background field in the symmetric phase generates an axial asymmetry in the scattering processes. We discuss the possible implications of such axial charge segregation for baryon number generation.

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I. INTRODUCTION

The possible existence of magnetic fields in the early universe has recently become the subject of intense research due to the many interesting cosmological implications that these entail [1]. For instance, magnetic fields can influence big bang nucleosynthesis (BBN), affecting the primordial abundance of light elements and the rate of expansion of the universe. The success of the standard BBN scenario can be used to set limits on the strength of the magnetic fields at this epoch. Moreover, at decoupling, long range magnetic fields can induce anisotropies in the cosmic microwave background radiation (CMBR). Temperature anisotropies from the Cosmic Background Explorer (COBE) results place an upper bound $B_0 \sim 10^{-9}$ G for homogeneous fields (B_0 refers to the intensity that the field would have today under the assumption of adiabatic decay due to the Hubble expansion) [2]. In the case of inhomogeneous fields their effect must be searched for in the Doppler peaks [3] and in the polarization of the CMBR [4]. The future CMBR satellite missions the Microwave Anisotropy Probe (MAP) and Planck may reach the required sensitivity for the detection of these last signals.

Another interesting cosmological consequence is the effect that primordial magnetic fields could have had on the dynamics of the electroweak phase transition (EWPT) at temperatures of the order of $T \sim 100$ GeV. In fact, it has been recently pointed out that, provided enough CP violation exists, large scale primordial magnetic fields can be responsible for a stronger first order EWPT [5–7] (see however Ref. [8]). The situation is similar to a type I superconductor where the presence of an external magnetic field modifies the nature of the superconducting phase transition due to the Meissner effect.

Recall that, for temperatures above the EWPT, the $SU(2) \times U(1)_Y$ symmetry is restored and the propagating, nonscreened vector modes that represent a magnetic field correspond to the $U(1)_Y$ group instead of to the $U(1)_{em}$ group, and are therefore properly called *hypermagnetic* fields.

In a previous work [9], we have shown, by using a simplified picture of a first order EWPT, that the presence of such fields also provides a mechanism, working in the same manner as the existence of additional CP violation within the SM, to produce an axial charge segregation in the scattering of fermions off the true vacuum bubbles. The asymmetry in the scattering of fermion axial modes is a consequence of the chiral nature of the fermion coupling to hypermagnetic fields in the symmetric phase. The simplification consisted in considering the limit of an infinitely thin bubble wall. This assumption allowed us to formulate the problem in terms of solving the Dirac equation with a position dependent fermion mass, proportional to a step function, this last being zero in the false phase and nonvanishing in the broken symmetry phase. This treatment rendered analytic solutions from where reflection and transmission coefficients for axial modes were straightforward computed.

In spite of the relative ease for the computation in such a scheme, there are two limitations related to the sudden change in the Higgs field profile that needed to be addressed. First, it is well known that the negative energy solutions of the Dirac equation become important in situations where the potential energy term changes over distances smaller than the particle's Compton wave length. Second, the height and width of the wall are typically related to each other in such a way that it is not entirely realistic to vary one without affecting the other.

In this paper we overcome the above limitations by allowing a finite width of the Higgs field profile. Working in the thin wall regime, we use the kink solution of the Higgs field to formulate and solve the Dirac equation in the presence of hypermagnetic fields. We compute explicitly transmission and reflection coefficients for the axial modes incident on the wall from the symmetric phase. Since these are related to the corresponding coefficients for fermions incident from the broken symmetry phase by *CPT* and unitarity, we find that the axial charge segregation still happens during fermion scattering of this wall. The existence of such asymmetric reflection for the axial modes provides a bias for baryon over antibaryon production. In the absence of hypermagnetic fields, this mechanism has been proposed and studied in Refs. [10–12] in extensions of the SM.

The outline of this work is as follows: In Sec. II, we briefly review how the kink solution for the spatial profile of the Higgs field is obtained from a finite temperature effective potential. In Sec. III, we set up the Dirac equation for fermions moving in this background Higgs field in the presence of an external hypermagnetic field. Section IV is devoted to a rather technical discussion about the solutions of this equation and their properties. In Sec. V, we use the above solutions to compute reflection and transmission coefficients for axial fermion modes moving from the symmetric phase toward the broken symmetry phase. We show that these coefficients differ for the two distinct helicity modes. Finally in Sec. VI, we conclude by looking out at the possible implications of such axially asymmetric fermion reflection and transmission.

II. KINK SOLUTION

To describe the EWPT, we start by writing the effective, finite temperature Higgs potential which, including all the one-loop effects and ring diagrams, looks like

$$V_{\text{eff}}(h, T) = \frac{\gamma}{2}(T^2 - T_c^2)h^2 - \delta T h^3 + \frac{\lambda}{4}h^4, \quad (1)$$

where $h = \sqrt{2}(H^\dagger H)^{1/2}$ is the strength of the SU(2) Higgs doublet H whose vacuum expectation value is given by

$$\langle H \rangle = \frac{v}{\sqrt{2}}. \quad (2)$$

The parameters γ , δ and λ have been computed perturbatively to one loop and can be expressed in terms of v , the SU(2) gauge boson masses and the top quark mass. Their explicit expressions can be found elsewhere (see for example Ref. [6]). δ is the parameter responsible for the first order nature of the phase transition. It is the parameter that gets enhanced in the presence of hypermagnetic fields. T_c is the critical temperature at which spinodal decomposition proceeds.

We can write the effective potential in a more transparent form [13] by introducing the dimensionless temperature ϑ and the dimensionless Higgs field strength φ ,

$$\vartheta = \frac{\lambda \gamma}{\delta^2} \left[1 - \left(\frac{T_c}{T} \right)^2 \right],$$

$$\varphi = \frac{\lambda}{\delta T} h, \quad (3)$$

in terms of which, the effective potential, Eq. (1), becomes

$$V_{\text{eff}}(\varphi) = \delta T \left(\frac{\delta T}{\lambda} \right)^3 \left(\frac{\vartheta}{2} \varphi^2 - \varphi^3 + \frac{1}{4} \varphi^4 \right). \quad (4)$$

For simplicity, we work in the approximation where the energy densities of both the unbroken and broken phases are degenerate. This happens for a value of $\vartheta = 2$. In this approximation, the phase transition is described by a one-dimensional solution for the Higgs field, called the *kink*, which separates the two phases. This is given by

$$\varphi(x) = 1 + \tanh(x), \quad (5)$$

where the dimensionless position coordinate x is

$$x = \frac{\delta T}{\sqrt{2\lambda}} z. \quad (6)$$

The parameter $\sqrt{2\lambda}/(\delta T)$ represents the width of the domain wall [14]. It can also be checked that this parameter becomes smaller in the presence of hypermagnetic fields.

In terms of the kink solution we can see that $x = -\infty$ represents the region outside the bubble, that is the region in the symmetric phase. Conversely, for $x = +\infty$, the system is inside the bubble, that is in the broken phase. The kink wall propagates with a velocity determined by its interactions with the surrounding plasma. This velocity can be anywhere between 0.1–0.9 the speed of light [15].

III. DIRAC EQUATION FOR AXIAL FERMIONS IN A BACKGROUND HYPERMAGNETIC FIELD

In the presence of an external magnetic field, we need to consider that fermion modes couple differently to the field in the broken symmetry and the symmetry restored phases.

For $z \leq 0$, the coupling is chiral. Let

$$\begin{aligned} \Psi_R &= \frac{1}{2}(1 + \gamma_5)\Psi \\ \Psi_L &= \frac{1}{2}(1 - \gamma_5)\Psi \end{aligned} \quad (7)$$

represent, as usual, the right and left-handed chirality modes for the spinor Ψ , respectively. Then, the equations of motion for these modes, as derived from the electroweak interaction Lagrangian, are

$$\begin{aligned} \left(i\partial - \frac{y_L}{2} g' A \right) \Psi_L - m(z) \Psi_R &= 0 \\ \left(i\partial - \frac{y_R}{2} g' A \right) \Psi_R - m(z) \Psi_L &= 0, \end{aligned} \quad (8)$$

where $y_{R,L}$ are the right and left-handed hypercharges corresponding to the given fermion, respectively, g' the $U(1)_Y$ coupling constant and we take $A^\mu = (0, \mathbf{A})$ representing a, not as yet specified, four-vector potential having nonzero components only for its spatial part, in the rest frame of the wall.

The set of Eqs. (8) can be written as a single equation for the spinor $\Psi = \Psi_R + \Psi_L$ by adding up the former equations

$$\left\{ i\partial - A \left[\frac{y_R}{4} g' (1 + \gamma_5) + \frac{y_L}{4} g' (1 - \gamma_5) \right] - m(z) \right\} \Psi = 0 \quad (9)$$

where the fermion mass $m(z)$ is proportional to the vacuum expectation value of the Higgs field. Hereafter, we explicitly work in the chiral representation of the gamma matrices where

$$\gamma^0 = \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}. \quad (10)$$

Within this representation, we can write Eq. (9) as

$$\{ i\partial - \mathcal{G} A_\mu \gamma^\mu - m(z) \} \Psi = 0, \quad (11)$$

where we have introduced the matrix

$$\mathcal{G} = \begin{pmatrix} \frac{y_L}{2} g' I & 0 \\ 0 & \frac{y_R}{2} g' I \end{pmatrix}. \quad (12)$$

We now look at the corresponding equation in the broken symmetry phase. For $z \geq 0$ the coupling of the fermion with the external field is through the electric charge e and thus, the equation of motion is simply the Dirac equation describing an electrically charged fermion in a background magnetic field, namely,

$$\{ i\partial - e A_\mu \gamma^\mu - m(z) \} \Psi = 0. \quad (13)$$

In the following section, we explicitly construct the solutions to Eqs. (11) and (13) with a constant magnetic field, requiring that these match at the interface $z=0$.

IV. SOLVING THE DIRAC EQUATION

Let us first find the solution to Eq. (11), namely, for fermions moving in the symmetric phase, $z \leq 0$. For this purpose, we look for a solution of the form

$$\Psi = \{ i\partial - A_\mu \gamma^\mu \mathcal{G} + m(z) \} \Phi. \quad (14)$$

Inserting this expression into Eq. (11), we obtain

$$\left\{ -\partial^2 - i\mathcal{G} \partial^\mu A_\mu - \frac{1}{2} \sigma^{\mu\nu} \mathcal{G} F_{\mu\nu} - 2i\mathcal{G} A_\mu \partial^\mu + \mathcal{G}^2 A_\mu A^\mu + i\gamma^\mu \partial_\mu m(z) \right\} \Phi = 0, \quad (15)$$

where, as usual,

$$\sigma^{\mu\nu} = \frac{i}{2} [\gamma^\mu, \gamma^\nu]$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \quad (16)$$

Given that the present analysis is carried out in the thin wall approximation, which in physical terms means that the spatial region over which the fermion mass changes is small compared to other relevant length scales such as the particle mean free path, our choice of external gauge fields and thus field strengths should try to capture this information, namely, that the change in the magnetic field strength occurs over a small spatial region. An exact treatment of the gauge fields should be to find the configuration that incorporates the boundary conditions imposed by the change in the Higgs profile. This treatment will render continuous gauge fields across the interface. An additional feature will be the generation of a component of the magnetic field directed along the transverse direction, since the longitudinal component of the gauge fields will vary along the longitudinal direction. This component will be localized also in the small region comprised by the phase boundary.

Notice then that when including this transverse component of the magnetic field into the Dirac equation, the separation of variables that factorizes longitudinal and transverse motion will not be possible. However, as long as this transverse field is confined to a small region and its strength is not too large to avoid capturing low energy fermions, it is a reasonable approximation to consider a constant field in the longitudinal direction since when the incident flux is not lost in modes captured on the wall, the probabilities of transmission and reflection depend on the particle currents computed in the asymptotic regions which in turn depend only on the fermion coupling to the external field, already constant in these regions.

To estimate the magnitude of such a transverse field able to capture fermions on the wall, let us consider the following classical argument. Equilibrium between the Lorentz force and the centrifugal force gives for the radius of the orbit for a particle trapped in the wall $R = p/(eB)$ where p is the particle's momentum, e is its charge and B the strength of the magnetic field. Taking R as the wall width λ , and since [see Eq. (27)], $B = b/(\lambda^2)$, then $b = p\lambda/e$. Taking $p \sim T$ (the momentum of a typical particle in thermal equilibrium) and since $\lambda \sim T^{-1}$ [see Eq. (6)] means that $b \sim 3$ which is already a very high value of the magnetic field. Particles with smaller momenta could however be trapped in this transverse component of the field. For the purposes of the present work, we

postpone the consequences of such external field configuration and consider a piece wise constant magnetic field.

For definiteness, let us consider the field $\mathbf{B}=B\hat{z}$ pointing along the \hat{z} direction. In this case, the vector potential \mathbf{A} can only have components perpendicular to \hat{z} and the solution to Eq. (15) factorizes as [16,17]

$$\Phi(t, \mathbf{x}) = \tau(x, y) \Phi(t, z). \quad (17)$$

We concentrate on the solution describing the motion of positive energy fermions perpendicular to the wall, i.e., along the \hat{z} axis. We thus look for stationary states, namely

$$\Phi(t, z) = e^{-iEt} \Phi(z). \quad (18)$$

Therefore, working in the Lorentz gauge, $\partial^\mu A_\mu = 0$, Eq. (15) becomes

$$\left\{ \frac{d^2}{dz^2} + i\gamma^3 \frac{dm(z)}{dz} + E^2 + iB\mathcal{G}\gamma^1\gamma^2 \right\} \Phi(z) = 0. \quad (19)$$

Notice that Eqs. (15) and (19) have the appropriate limit when $y_R = y_L = e$, corresponding to the description of fermions coupled with their electric charge to a background magnetic field [16].

We now expand $\Phi(z)$ in terms of the eigenspinors u_\pm^s ($s=1,2$) of γ^3 [18],

$$u_\pm^1 = \begin{pmatrix} 1 \\ 0 \\ \pm i \\ 0 \end{pmatrix}, \quad u_\pm^2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ \mp i \end{pmatrix}. \quad (20)$$

These spinors have the properties

$$\begin{aligned} \gamma^3 u_\pm^{1,2} &= \pm i u_\pm^{1,2} \\ \gamma^0 u_\pm^1 &= \mp i u_\mp^1 \\ \gamma^0 u_\pm^2 &= \pm i u_\mp^2 \\ \gamma^1 \gamma^2 u_\pm^1 &= -i u_\pm^1 \\ \gamma^1 \gamma^2 u_\pm^2 &= +i u_\pm^2 \\ \gamma_5 u_\pm^{1,2} &= u_\mp^{1,2}. \end{aligned} \quad (21)$$

Writing

$$\Phi(z) = \phi_+^1(z) u_+^1 + \phi_-^1(z) u_-^1 + \phi_+^2(z) u_+^2 + \phi_-^2(z) u_-^2 \quad (22)$$

and inserting this expression into Eq. (19), we obtain

$$\begin{aligned} \left[\frac{d^2}{dz^2} - \frac{dm}{dz} + (E^2 - m^2) + g'B \frac{(y_L + y_R)}{4} \right] \phi_+^1(z) \\ + g'B \frac{(y_L - y_R)}{4} \phi_-^1(z) = 0 \end{aligned} \quad (23)$$

$$\begin{aligned} \left[\frac{d^2}{dz^2} + \frac{dm}{dz} + (E^2 - m^2) + g'B \frac{(y_L + y_R)}{4} \right] \phi_-^1(z) \\ + g'B \frac{(y_L - y_R)}{4} \phi_+^1(z) = 0 \end{aligned}$$

and

$$\begin{aligned} \left[\frac{d^2}{dz^2} - \frac{dm}{dz} + (E^2 - m^2) - g'B \frac{(y_L + y_R)}{4} \right] \phi_+^2(z) \\ - g'B \frac{(y_L - y_R)}{4} \phi_-^2(z) = 0 \end{aligned} \quad (24)$$

$$\begin{aligned} \left[\frac{d^2}{dz^2} + \frac{dm}{dz} + (E^2 - m^2) - g'B \frac{(y_L + y_R)}{4} \right] \phi_-^2(z) \\ - g'B \frac{(y_L - y_R)}{4} \phi_+^2(z) = 0. \end{aligned}$$

Equations (23) and (24), represent, each, a set of two coupled second-order differential equations. The second set is obtained from the first one by changing B to $-B$. Consequently, Eqs. (23) and the corresponding functions and spinors with $s=1$ describe the motion of the spin components parallel to the magnetic field whereas Eqs. (24) and the functions and spinors with $s=2$, describe the motion of the spin components antiparallel to the magnetic field. Notice that in the limit when $y_R = y_L = e$, each set of equations decouple as is the case when describing the interaction of fermions with the magnetic field through their electric charge.

From now on, we focus on the set of Eqs. (23), since, as we have pointed out, the solutions to Eqs. (24) are obtained from those to Eqs. (23) by changing B to $-B$.

We now extract the dimensions writing the equations in terms of the dimensionless position coordinate x given in Eq. (6) and, furthermore, writing them in terms of the new variable

$$u = \frac{1 - \tanh(x)}{2} \quad (25)$$

obtaining, respectively,

$$\begin{aligned} \left[\frac{d^2}{du^2} + \frac{1-2u}{u(1-u)} \frac{d}{du} - \frac{\xi}{u(1-u)} + \frac{\epsilon^2}{4u^2(1-u)^2} - \frac{\xi^2}{u^2} \right. \\ \left. + \frac{g'b(y_L + y_R)}{16u^2(1-u)^2} \right] \phi_+^1(u) + \frac{g'b(y_L - y_R)}{16u^2(1-u)^2} \phi_-^1(u) = 0 \end{aligned}$$

$$\left[\frac{d^2}{du^2} + \frac{1-2u}{u(1-u)} \frac{d}{du} + \frac{\xi}{u(1-u)} + \frac{\epsilon^2}{4u^2(1-u)^2} - \frac{\xi^2}{u^2} + \frac{g'b(y_L+y_R)}{16u^2(1-u)^2} \right] \phi_-^1(u) + \frac{g'b(y_L-y_R)}{16u^2(1-u)^2} \phi_+^1(u) = 0, \quad (26)$$

where the parameters b and ξ are related to the magnetic field strength and the fermion mass by

$$b \equiv B \left(\frac{\delta T}{\sqrt{2\lambda}} \right)^{-2} \quad m(x) \equiv \left(\frac{\delta T}{\sqrt{2\lambda}} \right) \xi \varphi(x) \quad (27)$$

and furthermore, ξ represents twice the ratio of the fermion mass to the Higgs boson mass. ϵ is the energy parameter given by

$$\epsilon = \left(\frac{\delta T}{\sqrt{2\lambda}} \right)^{-1} E. \quad (28)$$

To further simplify Eqs. (26), we try the ansatz

$$\phi_{\pm}^1(u) = u^{\alpha_1} (1-u)^{\beta_1} \chi_{\pm}^1 \quad (29)$$

and examine the behavior of the resulting differential equations near the singular points $u=0$ and $u=1$. Assuming that the functions χ_{\pm} vary slowly near these singularities, we obtain the conditions

$$\alpha_1 = \begin{cases} \frac{i}{2} \sqrt{\epsilon^2 + \frac{g'}{2} b y_L - 4\xi^2} \equiv \alpha_1^L, \\ \frac{i}{2} \sqrt{\epsilon^2 + \frac{g'}{2} b y_R - 4\xi^2} \equiv \alpha_1^R, \end{cases} \quad (30)$$

$$\beta_1 = \begin{cases} \frac{i}{2} \sqrt{\epsilon^2 + \frac{g'}{2} b y_L} \equiv \beta_1^L, \\ \frac{i}{2} \sqrt{\epsilon^2 + \frac{g'}{2} b y_R} \equiv \beta_1^R. \end{cases} \quad (31)$$

Thus, to each pair of parameters, namely (α_1^L, β_1^L) and (α_1^R, β_1^R) , corresponds a pair of coupled differential equations which we write as

$$\left[u(1-u) \frac{d^2}{du^2} + [c^{1L} - (1 + a_{\mp}^{1L} + b_{\mp}^{1L})u] \frac{d}{du} - a_{\mp}^{1L} b_{\mp}^{1L} \right] \chi_{\pm}^{1L} = \pm f^{1L}(u), \quad (32)$$

$$\left[u(1-u) \frac{d^2}{du^2} + [c^{1R} - (1 + a_{\mp}^{1R} + b_{\mp}^{1R})u] \frac{d}{du} - a_{\mp}^{1R} b_{\mp}^{1R} \right] \chi_{\pm}^{1R} = f^{1R}(u),$$

where we have introduced the definitions for the functions f^{1L} and f^{1R} given by

$$f^{1L} \equiv \frac{\xi}{u(1-u)} (\chi_+^{1L} - \chi_-^{1L}) \quad (33)$$

$$f^{1R} \equiv -\frac{\xi}{u(1-u)} (\chi_+^{1R} + \chi_-^{1R})$$

together with that for the parameters $a_{\mp}^{1L,R}$, $b_{\mp}^{1L,R}$ and $c^{1R,L}$ given by

$$a_{\mp}^{1L,R} \equiv \alpha_1^{L,R} + \beta_1^{L,R} + \frac{1}{2} - \left| \xi_{\mp} \frac{1}{2} \right| \quad b_{\mp}^{1L,R} \equiv \alpha_1^{L,R} + \beta_1^{L,R} + \frac{1}{2} + \left| \xi_{\mp} \frac{1}{2} \right| \quad (34)$$

$$c^{1L,R} \equiv 2\alpha_1^{L,R} + 1,$$

and the parameter ξ is given by

$$\xi = \frac{g'}{2} b \frac{(y_L - y_R)}{8}. \quad (35)$$

The consistency of both sets of Eqs. (32) requires that, in the limit when $u \rightarrow 1$ ($z \rightarrow -\infty$),

$$\chi_+^{1L}(u) = A_1^L \chi_-^{1L} \quad \chi_+^{1R}(u) = -A_1^R \chi_-^{1R}, \quad (36)$$

with $A_1^{L,R}$ constants.

To solve Eqs. (32), we first notice that they are inhomogeneous hypergeometric differential equations. The solution appropriate to describe the motion of fermions in the symmetric phase is found by looking for the scattering states. For our purposes, these correspond to fermions incident toward and reflected from the wall. There are two types of such solutions; those coupled with y_L and those coupled with y_R . For an incident wave coupled with y_L (y_R), the fact that the differential equations mix up the solutions means that the reflected wave will also include a component coupled with y_R (y_L). Let us classify the solutions according to the type of wave that is incident toward the wall. For an incident wave coupled with y_L , which we call type (a), the most general solutions $\phi_{\pm}^{1(a)}(u)$ can be written as

$$\phi_{\pm}^{1(a)}(u) = (\phi_{\pm}^{1L})^I + (A_1^L)(\phi_{\pm}^{1L})^{II} \pm (A_1^R)(\phi_{\pm}^{1R})^{II} + (\phi_{\pm}^{1L})^{\text{part}} + (\phi_{\pm}^{1R})^{\text{part}}, \quad (37)$$

whereas for an incident wave coupled with y_R , which we call type (b), the most general solutions $\phi_{\pm}^{1(b)}(u)$ is written as

$$\phi_{\pm}^{1(b)}(u) = \pm(\phi_{\pm}^{1R})^I \pm (A_1^R)(\phi_{\pm}^{1R})^{II} + (A_1^L)(\phi_{\pm}^{1L})^{II} + (\phi_{\pm}^{1L})^{\text{part}} + (\phi_{\pm}^{1R})^{\text{part}}, \quad (38)$$

where the functions $(\phi_{\pm}^{1L,R})^I$ and $(\phi_{\pm}^{1L,R})^{II}$ are the linearly independent solutions corresponding to the solutions $(\chi_{\pm}^{1L,R})^I$ and $(\chi_{\pm}^{1L,R})^{II}$ of the homogeneous hypergeometric differential equation expressed as expansions around $u=1$, as appropriate for the symmetric phase,

$$(\phi_{\pm}^{1L,R})^I = u^{\alpha_1^{L,R}}(1-u)^{\beta_1^{L,R}} {}_2F_1(a_{\mp}^{1L,R}, b_{\mp}^{1L,R}, a_{\mp}^{1L,R} + b_{\mp}^{1L,R} + 1 - c^{1L,R}; 1-u) \quad (39)$$

$$(\phi_{\pm}^{1L,R})^{II} = u^{\alpha_1^{L,R}}(1-u)^{-\beta_1^{L,R}} {}_2F_1(c^{1L,R} - b_{\mp}^{1L,R}, c^{1L,R} - a_{\mp}^{1L,R}, c^{1L,R} - a_{\mp}^{1L,R} - b_{\mp}^{1L,R} + 1; 1-u).$$

$(\phi_{\pm}^{1L,R})^{\text{part}}$ are the solutions corresponding to the particular solutions $(\chi_{\pm}^{1L,R})^{\text{part}}$ of the inhomogeneous equations. The roman superindices I and II indicate the asymptotic behavior of the solution. I corresponds to an incoming wave (traveling to the right) whereas II correspond to a reflected wave (traveling to the left) in the symmetric phase, given the asymptotic behavior of the term $(1-u)^{\beta_1^{L,R}}$ when $u \rightarrow 1$ such that

$$(1-u)^{\pm\beta_1^{L,R}} \rightarrow e^{\pm 2\beta_1^{L,R}z}. \quad (40)$$

The particular solutions are expressed in terms of the functions f^{1L} and f^{1R} by the method of variation of parameters, and their explicit expressions are

$$\begin{aligned} (\phi_{\pm}^{1R})^{\text{part}}(u) &= -\frac{1}{2\alpha_1^R} \int_1^u \frac{(\phi_{\pm}^{1R}(s))^I (\phi_{\pm}^{1R}(u))^{II} - (\phi_{\pm}^{1R}(s))^{II} (\phi_{\pm}^{1R}(u))^I}{s^{-\alpha_1^R}(1-s)^{-\beta_1^R}} f^{1R}(s) ds \\ (\phi_{\pm}^{1L})^{\text{part}}(u) &= \mp \frac{1}{2\alpha_1^L} \int_1^u \frac{(\phi_{\pm}^{1L}(s))^I (\phi_{\pm}^{1L}(u))^{II} - (\phi_{\pm}^{1L}(s))^{II} (\phi_{\pm}^{1L}(u))^I}{s^{-\alpha_1^L}(1-s)^{-\beta_1^L}} f^{1L}(s) ds. \end{aligned} \quad (41)$$

To determine the solutions, we need knowledge of the functions $f^{1L,R}$, which in turn are given self-consistently by Eqs. (33). By substituting the formal solutions $\chi_{\pm}^{1(a,b)}(u)$ into Eqs. (33), this self-consistency is expressed in terms of integral equations satisfied by $f^{1L,R}$, given explicitly by

$$f^{1L,R}(u) = \rho^{1L,R}(u) + \frac{\zeta}{2\alpha_1^{R,L}} \int_1^u K^{1R,L}(u,s) f^{1L,R}(s) ds, \quad (42)$$

where we have introduced the functions $\rho^{1R,L}$ given by

$$\rho^{1R}(u) = -\frac{\zeta}{u(1-u)} \{(\chi_+^{1R})^{II} - (\chi_-^{1R})^{II}\} A_1^R \quad (43)$$

$$\rho^{1L}(u) = \frac{\zeta}{u(1-u)} \{[(\chi_+^{1L})^I - (\chi_-^{1L})^I] + [(\chi_+^{1L})^{II} - (\chi_-^{1L})^{II}] A_1^L\}$$

and $K^{1R,L}$ given explicitly by

$$\begin{aligned} K^{1R}(u,s) &= \frac{1}{u^{\alpha_1^R+1}(1-u)^{\beta_1^R+1} s^{-\alpha_1^R}(1-s)^{-\beta_1^R}} \\ &\times [(\phi_+^{1R}(u))^{II} (\phi_+^{1R}(s))^I - (\phi_+^{1R}(u))^I (\phi_+^{1R}(s))^{II} \\ &+ (\phi_-^{1R}(u))^{II} (\phi_-^{1R}(s))^I - (\phi_-^{1R}(u))^I (\phi_-^{1R}(s))^{II}] \end{aligned} \quad (44)$$

$$\begin{aligned} K^{1L}(u,s) &= \frac{-1}{u^{\alpha_1^L+1}(1-u)^{\beta_1^L+1} s^{-\alpha_1^L}(1-s)^{-\beta_1^L}} \\ &\times [(\phi_+^{1L}(u))^{II} (\phi_+^{1L}(s))^I - (\phi_+^{1L}(u))^I (\phi_+^{1L}(s))^{II} \\ &+ (\phi_-^{1L}(u))^{II} (\phi_-^{1L}(s))^I - (\phi_-^{1L}(u))^I (\phi_-^{1L}(s))^{II}]. \end{aligned}$$

The solution to Eqs. (42) is found numerically [19].

We now turn to finding the solution to Eq. (13), namely, for fermions moving in the broken symmetry phase, $z \geq 0$. This time, we look for a solution of the form

$$\Psi = \{i\partial - eA_{\mu}\gamma^{\mu} + m(z)\}\Phi. \quad (45)$$

Let us continue looking only at solutions type 1. By a procedure similar to that leading to Eqs. (26), the corresponding

expressions for the functions $\phi_{\pm}^1(z)$, representing a transmitted wave moving to the right in the broken symmetry phase become [16]

$$\phi_{\pm}^1(u) = B_{1\pm} u^{\alpha_1} (1-u)^{\beta_1} {}_2F_1(a_{\mp}^1 + 1 - c^1, b_{\mp}^1 + 1 - c^1, 2 - c^1; u), \quad (46)$$

with $B_{1\pm}$ constants and where the hypergeometric function ${}_2F_1$ is expressed as an expansion around $u=0$, as is appropriate for this region. The parameters a_{\mp}^1 , b_{\mp}^1 and c^1 are given by

$$\begin{aligned} a_{\mp}^1 &= \alpha_1 + \beta_1 + \frac{1}{2} - \left| \xi_{\mp} \frac{1}{2} \right| \\ b_{\mp}^1 &= \alpha_1 + \beta_1 + \frac{1}{2} + \left| \xi_{\mp} \frac{1}{2} \right| \\ c^1 &= 2\alpha_1 + 1 \end{aligned} \quad (47)$$

with

$$\begin{aligned} \alpha_1 &= \frac{i}{2} \sqrt{\epsilon^2 + g'b - 4\xi^2} \\ \beta_1 &= \frac{i}{2} \sqrt{\epsilon^2 + g'b}. \end{aligned} \quad (48)$$

Notice that since in the broken symmetry phase there should not be a propagating component corresponding to the Z^0 field [5,6], the parameter representing the magnetic field strength b' is related to b and Weinberg's angle θ_W by

$$b' = \frac{b}{\cos \theta_W}, \quad (49)$$

which in turn implies that the coupling of the fermion with the magnetic field is given by

$$eb' = g'b. \quad (50)$$

The complete solution to the problem is found by matching the functions $\phi_{\pm}^{1(a),(b)}(u)$ and $\phi_{\pm}^1(u)$ as well as their derivatives across the interface at $u=1/2$. These conditions represent four algebraic complex equations that determine the four complex constants $A_1^{L,R}$ and $B_{1\pm}$.

V. TRANSMISSION AND REFLECTION COEFFICIENTS

The fact that the amplitudes for the axial modes in the symmetric phase, Eqs. (37) and (38) are not the same, means that there is the possibility of building an axial asymmetry during the scattering of fermions off the wall. To quantify the asymmetry, we need to compute the corresponding reflection and transmission coefficients. These are built from the reflected, transmitted and incident currents of each type. Recall that for a given spinor wave function Ψ , the current normal to the wall is given by

$$J = \Psi^\dagger \gamma^0 \gamma^3 \Psi. \quad (51)$$

The currents need be computed in the asymptotic regions far away from the wall where the amplitudes represent plane waves with well defined direction of motion.

We now prepare the incident fermion from the symmetric phase in such a way that when coupled with a given *chirality* [left-handed for waves type (a), right-handed for waves type (b)], it corresponds to the same *helicity*. Since in the symmetric phase the fermion mass is asymptotically zero, both the chirality and helicity operators can be simultaneously defined and their eigenvalues coincide. This is no longer the case when the fermion moves in the broken symmetry phase where its mass is different from zero. Nevertheless, since scattering off the wall does not change the direction of the fermion spin (modes 1 and 2 evolve independently) the fermion helicity is preserved during transmission and reversed upon reflection.

For left-handed incoming waves, the incident current J_{inc}^l (lower case indexes l and r denote helicity modes) is thus given by

$$J_{\text{inc}}^l = 4|i\epsilon + 2\beta_2^L|^2 \quad (52)$$

whereas the reflected and transmitted currents J_{ref}^r , J_{tra}^l are given respectively by

$$\begin{aligned} J_{\text{ref}}^r &= 4\{|A_2^L|^2 - i\epsilon + 2\beta_2^L|^2 - |A_2^R|^2|i\epsilon + 2\beta_2^R|^2\} \\ J_{\text{tra}}^l &= \{|B_2 + [2(\xi - \alpha_2) - i\epsilon] - B_2 - [2(\xi + \alpha_2) + i\epsilon]|^2 \\ &\quad + |B_2 + [2(\xi - \alpha_2) + i\epsilon] - B_2 - [2(\xi + \alpha_2) - i\epsilon]|^2\}. \end{aligned} \quad (53)$$

On the other hand, for right-handed incoming waves, the incident current J_{inc}^r is thus given by

$$J_{\text{inc}}^r = 4|i\epsilon + 2\beta_1^R|^2 \quad (54)$$

and the reflected and transmitted currents J_{ref}^l , J_{tra}^r are given respectively by

$$\begin{aligned} J_{\text{ref}}^l &= 4\{|A_1^R|^2 - i\epsilon + 2\beta_1^R|^2 - |A_1^L|^2|i\epsilon + 2\beta_1^L|^2\} \\ J_{\text{tra}}^r &= \{|B_1 + [2(\xi - \alpha_1) - i\epsilon] + B_1 - [2(\xi + \alpha_1) + i\epsilon]|^2 \\ &\quad - |B_1 + [2(\xi - \alpha_1) + i\epsilon] + B_1 - [2(\xi + \alpha_1) - i\epsilon]|^2\}. \end{aligned} \quad (55)$$

For a left-handed incident particle, the reflection and transmission coefficients are given as the ratios of the corresponding reflected and transmitted currents, to the incident one, respectively, projected along a unit vector normal to the wall. These are

$$\begin{aligned} R_{l \rightarrow r} &= -J_{\text{ref}}^r / J_{\text{inc}}^l \\ T_{l \rightarrow l} &= J_{\text{tra}}^l / J_{\text{inc}}^l. \end{aligned} \quad (56)$$

The corresponding coefficients for the axially conjugate process are

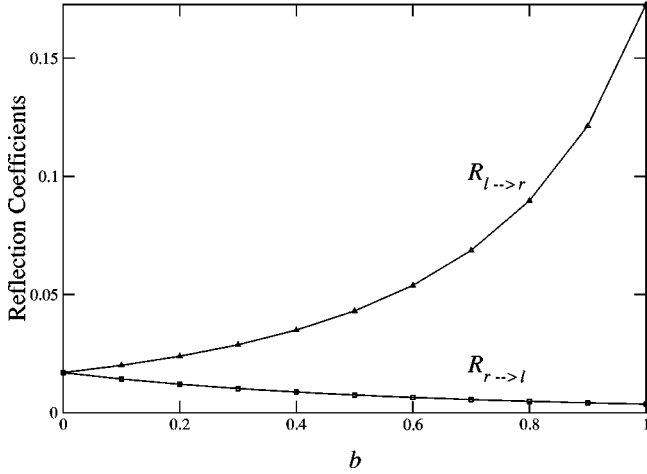


FIG. 1. Coefficients $R_{l \to r}$ and $R_{r \to l}$ as a function of the magnetic field parameter b for $\xi=3.5$, $\epsilon=7.03$, $y_R=4/3$, $y_L=1/3$. The value for the $U(1)_Y$ coupling constant is taken as $g'=0.344$, corresponding to the EWPT epoch. The dots represent the computed values.

$$\begin{aligned} R_{r \to l} &= -J_{\text{ref}}^r / J_{\text{inc}}^r \\ T_{r \to r} &= J_{\text{tra}}^r / J_{\text{inc}}^r. \end{aligned} \quad (57)$$

Figure 1 shows the coefficients $R_{l \to r}$ and $R_{r \to l}$ as a function of the magnetic field parameter b for a value of twice the ratio of fermion to Higgs boson mass $\xi=3.5$, an energy parameter $\epsilon=7.03$, hypercharge values $y_R=4/3$, $y_L=1/3$ and for a value of $g'=0.344$, as appropriate for the EWPT epoch. Notice that when $b \rightarrow 0$, these coefficients approach each other and that the difference grows with increasing field strength.

Figure 2 shows the reflection and transmission coefficients as a function of the energy parameter ϵ scaled by twice the height of the barrier 2ξ . Figure 2(a) shows the coefficients $R_{l \to r}$ and $T_{l \to l}$ and Fig. 2(b) the coefficients $R_{r \to l}$ and $T_{r \to r}$ for $b=0.5$ and $\xi=3.5$, $y_R=4/3$, $y_L=1/3$, $g'=0.344$. Since the solutions in Eqs. (46) are computed assuming that the transmitted waves are not exponentially damped, their energies have to be taken such that the parameters $\alpha_{1,2}$ are imaginary which in turn implies that for waves type 1, $\epsilon \geq \sqrt{4\xi^2 - g'b}$ whereas for waves type 2, $\epsilon \geq \sqrt{4\xi^2 + g'b}$. It can be checked that $R_{r \to l} + T_{r \to r} = 1$ and $R_{l \to r} + T_{l \to l} = 1$ within the numerical precision of the calculation, which means that the analysis respects unitarity.

VI. CONCLUSIONS

In this paper we have derived and solved the Dirac equation for fermions scattering off a first order EWPT bubble wall with a finite width in the presence of a magnetic field directed along the fermion direction of motion. In the symmetric phase, the fermions couple chirally to the magnetic field, which receives the name of *hypermagnetic*, given that it belongs to the $U(1)_Y$ group. We have shown that the chiral nature of this coupling implies that it is possible to build an

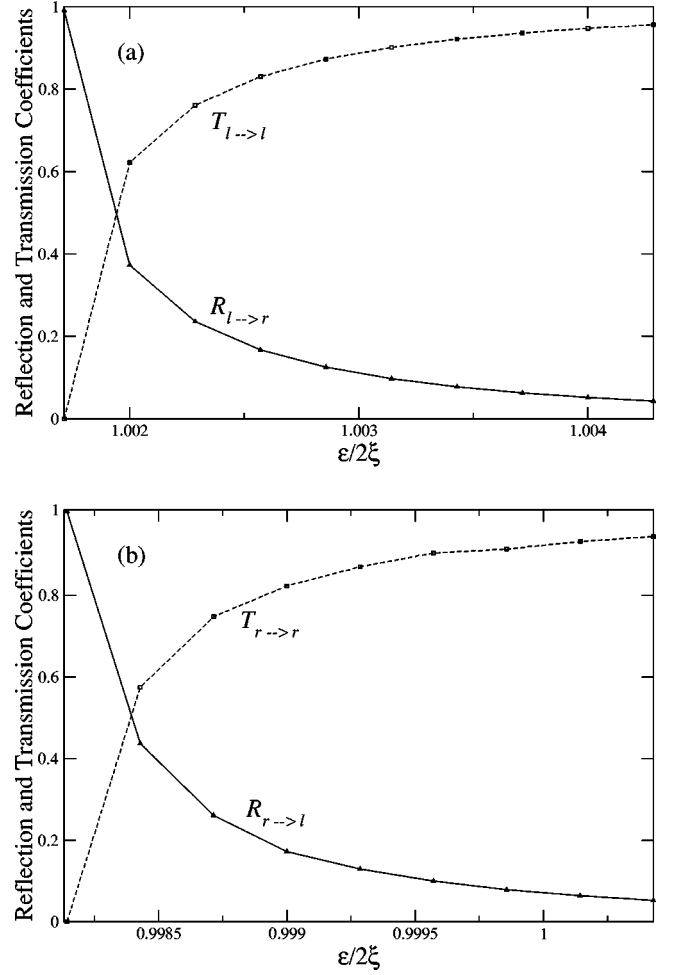


FIG. 2. Reflection and transmission coefficients as a function of the energy parameter ϵ scaled by twice the height of the barrier 2ξ for $b=0.5$ and $\xi=3.5$, $y_R=4/3$, $y_L=1/3$, $g'=0.344$. Figure 2(a) (upper panel) shows the coefficients for incident, left-handed helicity modes and Fig. 2(b) (lower panel) for incident, right-handed helicity modes. In both figures, the dots represent the computed values.

axial asymmetry during the scattering of fermions off the wall. We have computed reflection and transmission coefficients showing explicitly that they differ for left and right-handed incident particles from the symmetric phase. The results of this more realistic, albeit numerical calculation where we allow for a finite wall width are in qualitative and quantitative agreement with those previously found in Ref. [9], where the wall was modeled as a step function.

It could be thought that the asymmetric reflection found in this work could be washed out when considering the averaging over the different angles of incidence of the fermion flux. This is not the case as we proceed to show. Let us first look at the situation in which the direction of the magnetic field is reversed with respect to the case studied here. This a physically relevant scenario since during the phase transition, fermions are scattered on opposite sides of the bubbles and if the sign of the asymmetry would depend on the direction of the magnetic field with respect to the direction of fermion incidence, then the building of an axial charge density in one side of the bubble would compensate the building of this

charge on the other side, thereby canceling the effect. However, it is easy to convince oneself that this is not the case. By looking at Eqs. (23) and (24), we see that changing B to $-B$ interchanges one set of equations with the other, leaving intact the coupling. Physically this is also easy to understand since the fermion coupling with the external field is through its spin. Changing the direction of the field exchanges the role of each spin component but since each chirality mode contains both spin orientations, it does not affect the final probabilities.

Now suppose that the original direction of motion of the fermion is not parallel to the direction of the magnetic field and therefore its velocity vector contains a component perpendicular to the direction of the field. In this case, due to the Lorentz force, the particle circles around the field lines maintaining its velocity along the direction of the field. The motion of the particle is thus described as an overall displacement along the field lines superimposed to a circular motion around these lines. In the three dimensional quantum mechanical treatment of the problem, these circles correspond to the different Landau levels. We see that the originally different angles of incidence all result in the same overall direction of incidence. Nonetheless, it is certainly true that these circular trajectories could be regarded as the paths where the wave function of the particle picks up a phase in the same manner as in the Aharonov-Bohm effect. However, since there is no definite phase relation of the incident fermions, these phases have to be regarded as randomly distributed. Thus, the addition of the wave functions at the interference point (minus infinity for the reflected waves and plus infinity for the transmitted waves) has to be done incoherently which precludes any possible destructive effect of these phases on the overall particle fluxes.

We also emphasize that, under the very general assumptions of CPT invariance and unitarity, the total axial asymmetry (which includes contributions both from particles and antiparticles) is quantified in terms of the particle (axial) asymmetry. Let ρ_i represent the number density for species i . The net densities in left-handed and right-handed axial charges are obtained by taking the differences $\rho_L - \rho_{\bar{L}}$ and $\rho_R - \rho_{\bar{R}}$, respectively. It is straightforward to show [12] that CPT invariance and unitarity imply that the above net densities are given by

$$\begin{aligned}\rho_L - \rho_{\bar{L}} &= (f^s - f^b)(R_{r \rightarrow l} - R_{l \rightarrow r}) \\ \rho_R - \rho_{\bar{R}} &= (f^s - f^b)(R_{l \rightarrow r} - R_{r \rightarrow l}),\end{aligned}\quad (58)$$

where f^s and f^b are the statistical distributions for particles or antiparticles (since the chemical potentials are assumed to be zero or small compared to the temperature, these distributions are the same for particles or antiparticles) in the symmetric and the broken symmetry phases, respectively. From Eq. (58), the asymmetry in the axial charge density is finally given by

$$(\rho_L - \rho_{\bar{L}}) - (\rho_R - \rho_{\bar{R}}) = 2(f^s - f^b)(R_{r \rightarrow l} - R_{l \rightarrow r}). \quad (59)$$

This asymmetry, built on either side of the wall, is dissociated from nonconserving baryon number processes and can subsequently be converted to baryon number in the broken symmetry phase where sphaleron induced transitions are taking place with a large rate. This mechanism receives the name of *nonlocal baryogenesis* [10,12,11,20] and, in the absence of the external field, it can only be realized in extensions of the SM where a source of CP violation is introduced *ad hoc* into a complex, space-dependent phase of the Higgs field during the development of the EWPT [21].

Since another consequence of the existence of an external magnetic field is the lowering of the barrier between topologically inequivalent vacua [22], due to the sphaleron dipole moment, the use of the mechanism discussed in this work to possibly generate a baryon asymmetry is not as straightforward. Nonetheless, if such primordial fields indeed existed during the EWPT epoch and the phase transition was first order, as is the case, for instance, in minimal extensions of the SM, the mechanism advocated in this work has to be considered as acting in the same manner as a source of CP violation that can have important consequences for the generation of a baryon number. These matters will be the subject of an upcoming work [23].

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